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Thermal History of the Universe (Cont'd):

Assuming that the universe contains relativistic particles (as happens in very early times), the total energy density is given by:

$$\rho = g_* \frac{\pi^2}{30} T^4 \qquad g_* = g_B + \frac{7}{8} g_F$$

Then, from the first Friedmann equation, we have:

$$H^2 = \frac{8\pi G}{3} \rho = \frac{\rho}{3 M_P^2} \qquad M_P \equiv \left(\frac{1}{8\pi G}\right)^{1/2} = 2.4 \times 10^{18}\ \text{GeV}$$

M_P: reduced Plank mass

$$H^2 = g_* \frac{\pi^2}{90} \frac{T^4}{M_P^2}$$

Thus:

$$H = \left(g_* \frac{\pi^2}{90}\right)^{1/2} \frac{T^2}{M_P} \qquad (I)$$

In a radiation dominated universe, $H \propto T^2$. This also gives a relation between time and temperature:

$$H = \frac{1}{2t} \Rightarrow t = \frac{1}{2\left(g_* \frac{\pi^2}{90}\right)^{1/2}} \frac{M_P}{T^2}$$

In general, g_* is a function of temperature because at a given temperature T only those degrees of freedom that have a mass $m \ll T$ are relativistic. Gunting g_* in the early universe requires a knowledge of all elementary particles that exist. The currently know particles and their interactions are described by the "Standard Model of Particle Physics". Here, we give a short review of the elementary particles and their properties according to the standard model.

In the standard model, the fundamental particles come in three groups:

1- Building blocks of matter: these are spin-$\frac{1}{2}$ fermions,

2- Mediators of the forces: these are spin-1 bosons.

3- The Higgs particle: this is a spin-0 boson.

Let us describe these three groups in more detail.

1- The building blocks of matter are of two types:

(a) Quarks: they have all the three interactions (electromagnetism, weak, and strong). The nucleons (protons and neutrons) are bound states of three quarks.

$$\begin{array}{ll} q=+\frac{2}{3} \\ q=-\frac{1}{3} \end{array} \begin{bmatrix} u \\ d \end{bmatrix} \begin{array}{ll} \text{up quark} & m_u \sim 2.3 \text{ MeV} \\ \text{down quark} & m_d \sim 4.8 \text{ MeV} \end{array}$$

Proton consists of two up quarks and one down quark (hence electric charg +1), while neutron consists of two down quarks and one up quark (hence 0 electric charge).

There are also two heavier copies of quarks with the same charges as u, d quarks:

$$\begin{array}{ll} q=+\frac{2}{3} \\ q=-\frac{1}{3} \end{array} \begin{bmatrix} c \\ s \end{bmatrix} \begin{array}{ll} \text{charm quark} & m_c \sim 1.3 \text{ GeV} \\ \text{strange quark} & m_s \sim 95 \text{ MeV} \end{array}$$

$$\begin{array}{ll} q=+\frac{2}{3} \\ q=-\frac{1}{3} \end{array} \begin{bmatrix} t \\ b \end{bmatrix} \begin{array}{ll} \text{top quark} & m_t \sim 170 \text{ GeV} \\ \text{bottom quark} & m_b \sim 4.6 \text{ GeV} \end{array}$$

No free quarks have been observed at low energies (or long distances). The reason being that the strong force binds them together. The quantum number that is related to the strong interactions is called "color" (just as the electric charge that is related to the electromagnetic interactions). All quarks come in three different colors.

In consequece, each quark represents 12 fermionic degrees of freedom $12 = 2 \times 2 \times 3$. One factor of 2 is due to spin, another factor of 2 is due to antiparticles, and factor of 3 is associated with the color. Altogether, there are 72 fermionic degrees of freedom represented by quarks:

$$g_F^{\text{quarks}} = \underbrace{6}_{\text{6 quarks}} \times \underbrace{2}_{\text{spin-}\frac{1}{2}} \times \underbrace{2}_{\text{particles + antiparticles}} \times \underbrace{3}_{\text{color}} = 72$$

(b) Leptons: they have only electromagnetic and weak interactions. The leptons have either electric charge -1 or are neutral. The lightes charged lepton (which is also stable) is the electron that is bound to nuclei by the electromagnetic force to form atoms. Like quarks, the leptons come in three copies (called family/flavor/generation):

$$q=0 \quad \begin{bmatrix} \nu_e \\ e \end{bmatrix} \quad \begin{matrix} \text{electron neutrino} \\ \text{electron} \end{matrix} \qquad m_e \sim 0.5\ \text{MeV}$$
$$q=-1$$

$$q=0 \quad \begin{bmatrix} \nu_\mu \\ \mu \end{bmatrix} \quad \begin{matrix} \text{muon neutrino} \\ \text{muon} \end{matrix} \qquad m_\mu \sim 105\ \text{MeV}$$
$$q=-1$$

$$q=0 \quad \begin{bmatrix} \nu_\tau \\ \tau \end{bmatrix} \quad \begin{matrix} \text{tau neutrino} \\ \text{tau} \end{matrix} \qquad m_\tau \sim 1.8\ \text{GeV}$$
$$q=-1$$

The three neutrinos ν_e, ν_μ, ν_τ have only weak interactions. From atmospheric neutrino oscillations, there exists a mass scale $m_{atm} \sim 0.05\ \text{eV}$ for the difference in neutrino masses.

The combined limits from cosmology and laboratory experiments set an upper bound of $\sim 0.5\,eV$ on the sum of all neutrino masses, but the individual neutrino masses are not known. Each charged lepton represents 4 degrees of freedom (two from spin and two for antiparticles). On the other hand, each neutrino represents two degrees of freedom that have weak interactions: one orientation of spin for particles and the other orientation for antiparticles. Therefore, altogether, there are 18 fermionic degrees of freedom associated with leptons:

$$g_F^{\text{leptons}} = \underbrace{3}_{\substack{\text{3 charged}\\\text{leptons}}} \times \underbrace{2}_{\text{spin }\frac{1}{2}} \times \underbrace{2}_{\substack{\text{particles}\\+\\\text{antiparticles}}} + \underbrace{3 \times 2}_{\text{3 neutrinos}} = 18$$

2- Mediators of the three interactions (electromagnetic, weak, and strong) are spin-1 particles. The most familiar one is the photon γ that is the messanger of electromagnetic interactions:

$q=0$ γ $m_\gamma = 0$

Being a massless particle, it represents two degrees of freedom (the two transverse polarizations). There are three spin-1 particles that are messangers of the weak interactions:

$q=0$ Z $m_Z \sim 90$ GeV

$q=\pm 1$ $W^\pm$ $m_W \sim 80$ GeV

The fact that $W^\pm$, Z are massive, explains why the weak interactions are short range. The exchange of $W^\pm$ and Z particles gives rise to Yukawa potentials as opposed to the exchange of photons that results in Coulomb potential with infinite range. Since $W^\pm$, Z are massive, each of them represents three degrees of freedom of a spin-1 particle:

two transverse and one longitudinal polarizations.

Finally, we have the messangers of the strong interactions called gluons G. There are 8 gluons, which are massless. Hence, just like photons, each of the gluons represents two degrees of freedom. However, we do not observe free gluons at low energies because they get bound by strong interactions.

$q=0 \qquad G \qquad m_G=0 \qquad (8\ G\text{'s})$

Together, the messangers of the fundamental interactions represent 27 degrees of freedom:

$$g_B^{\text{messangers}} = \underbrace{2}_{\substack{\text{photon} \\ \text{(2 polarizations)}}} + \underbrace{3}_{W^\pm,\ Z} \times \underbrace{3}_{\substack{\text{three} \\ \text{polarizations}}} + \underbrace{2}_{\text{two polarizations}} \times \underbrace{8}_{\text{gluons}}$$

3- The Higgs particle: it is a spin-0 boson that is electrically neutral:

$q=0 \qquad h \qquad m_h \sim 126\ \text{GeV}$

The discovery of Higgs particle in 2012 closed the list of elementary particles in the standard model. The role of Higgs is to give mass to other particles of the standard model in a theoretically consistent manner. The Higgs represents one bosonic degree of freedom:

$$g_B^{Higgs} = 1$$

As a result, at temperature $T \gg 170\,GeV$ (note that the top quark is the heaviest particle in the standard model) we have:

$$g_*^{SM} = g_B + \frac{7}{8} g_F = g_B^{Higgs} + g_B^{messangers} + \frac{7}{8}\left(g_F^{quarks} + g_F^{leptons}\right) \Rightarrow$$

$$g_*^{SM} = (1+27) + \frac{7}{8}(72+18) \Rightarrow g_*^{SM} = 106.75 \qquad (\text{II})$$

At such high temperature, from Eq. (I), we find:

$$H = \left(106.75 \times \frac{\pi^2}{30}\right)^{\frac{1}{2}} \frac{T^2}{M_P} \Rightarrow H \sim 3 \frac{T^2}{M_P}$$

We can also write the t-T relation in this case:

$$t \sim \frac{1}{6} \frac{M_p}{T^2}$$

If $T \sim M_p$, then we find $t = t_{planck} \sim 10^{-43}$ sec.

Expansion of the universe results in decrease in the temperature T. Particles that are massive will have a transition to the non-relativistic regime when $T \lesssim m$. As a result, the number of relativistic degrees of freedom will decrease. The question is how the non-relativistic degrees of freedom contribute to the energy density of the universe and hence its expansion.

To elaborate on this let us consider the standard model particles. Starting from very high temperatures, the first particle that becomes non-relativistic is the heaviest among the standard model particles, i.e., the top quark. If top quarks retain their number density, which is the same as relativistic particles, they will soon dominate the energy density since $\frac{\rho_{non-rel}}{\rho_{rel}} \propto a$.

Eq. (I) will then not be valid since it only takes the contribution from relativistic degrees of freedom into account. However, top quark is unstable. This implies that its decay will reduce the number density when $T < m_t$. The reason being that at these temperatures the inverse decay is suppressed kinematically as relativistic particles in the thermal bath do not have enough energy to make a top quark via inverse decay. The question is whether the decay happens sufficiently quickly so that top quarks practically disappear before the universe expands significantly. This will be the case if $\Gamma_t \gg H$, Γ_t being the decay width of the top quark, in which case the number density of top quarks gets so small that they will not contribute to the subsequent expansion of the universe. The efficiency of the decay can be verified by comparing Γ_t with H at a temperature $T \sim m_t$ (when inverse decays become inefficient);

$\Gamma_t \sim 2$ GeV (from standard model)

$$H_{(T\sim m_t)} \sim \frac{3\, m_t^2}{M_P}$$

Inserting the numerical values for m_t and M_P, we find that $\Gamma_t \gg H_{(T\sim m_t)}$. This confirms that top quarks decay extremely fast as soon as T drops below m_t.

In fact, this is the case for all unstable massive particles in the standard model (Higgs, $W^\pm$ and Z particles, etc). They decay almost instantly once they become non-relativistic. Thus they do not make further contributions to the energy density.

Only particles that are relativistic at a temperature T contribute to the Hubble expansion rate at that temperature.

Therefore g_* decreases monotonically in adiabatic fashion as the universe keeps expanding.

To give an example, let us consider $T \sim O(\text{MeV})$. At such a temperature, the only relativistic degrees of freedom are photons, neutrinos and electrons (plus their antiparticles), while heavier particles have completely decayed and disappeared. Thus:

$$g_* = \underbrace{2}_{\text{photons}} + \frac{7}{8} \times (\underbrace{4}_{\substack{\text{electrons}\\ \text{and}\\ \text{positrons}}} + \underbrace{3 \times 2}_{\substack{\text{3 flavors}\\ \text{of}\\ \text{neutrinos}}}) = 10.75$$

$$H \sim \frac{T^2}{M_P} \qquad t \sim \frac{1}{2} \frac{M_P}{T^2} \qquad (t \sim 1\,\text{sec} \equiv T \sim 1\,\text{MeV})$$

It is seen that the change in g_* from 106.75 at $T \gg m_t$ to 10.75 at $T \sim 1$ MeV, does not lead to a drastic change in the prefactor that relates H to $\frac{T^2}{M_P}$ (a chang from ~ 3 to ~ 1).

An important comment is in order at this point. There are some massive standard model particles that are stable: electrons and protons. The same holds for particles that make dark matter in the universe (these are new particles that require physics beyond the standard model). A question arises as what happens to the particles once temperature drops below their mass. It is true that they will not decay due to their stable nature. However, they can undergo pair annihilation with their antiparticles. Similar to inverse decay (for unstable particles) inverse annihilations are kinematically suppressed at temperatures lower than the particle mass. Annihilations will quickly decrease the number density of massive particles that become non-relativistic. However, unlike decays, annihilations become inefficient at low number densities (as discussed before). Therefore, even if $\Gamma_{ann} \gg H$

at initial stages when $T \lesssim m$, it will eventually switch to the opposite limit $\Gamma_{ann} \lesssim H$. At this time, the contribution of non-relativistic particles to the energy density is very small and negligible. However, since these particles have not completely disappeared as happens for unstable particles, they will eventually dominate the energy density (recall that $\frac{\rho_{non-rel}}{\rho_{rel}} \propto a$).

This is exactly the reason why the universe enters a matter-dominated phase after an early radiation-dominated phase. The stable particles that dominate the universe at $t_{eq} \sim 50{,}000\ yr$ are dark matter (mainly) and baryons (a subdominant component).